Uxendon Manor Primary School

Concrete, Pictorial & Abstract (CPA) Calculation Policy

This policy contains the key procedures that are to be taught throughout the school. It has been written to ensure consistency and progression throughout the school. We are aiming to get each child to show fluency, reasoning and problem solving skills from EYFS – Year 6.

- Although the main focus of this policy is showing the core **Concrete, Pictorial and Abstract** ways of solving Maths problems, it is important to recognise that the ability to calculate mentally lies at the heart of numeracy.
- Mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing.
- **Written recording** both helps children to clarify their thinking and supports and extends the development of more ***fluent*** and sophisticated mental strategies.
- Children are encouraged to use **the most efficient method for them**, making sure they use ones they have a clear understanding of.
- **The long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task.** They should do this by always asking themselves:

  - **'Do I need to use manipulatives to help me?'**
  - **'Can I do this using drawings or jottings?'**
  - **'Do I need to use a written method?'**
  - **'Can I do this in my head?'**

Stem sentences are in red, these are to help children embed their learning and an additional list at the back of the document

Compiled by G Wagjiani adapted from White Rose - May 2019.

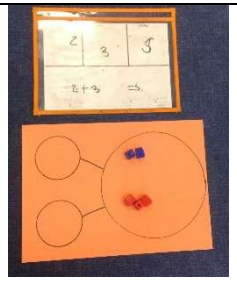
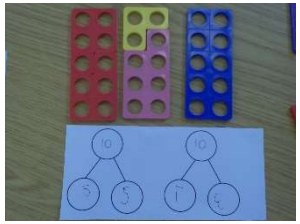
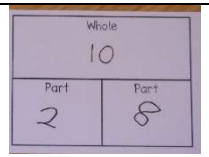
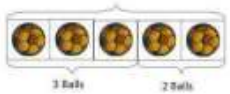
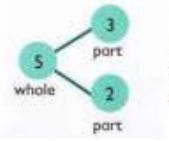
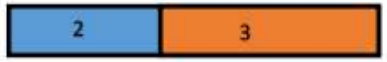
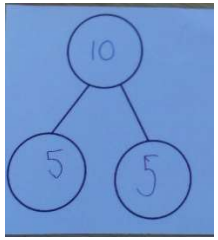
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Addition

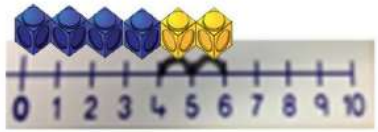
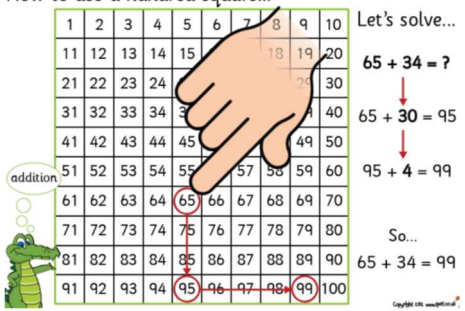
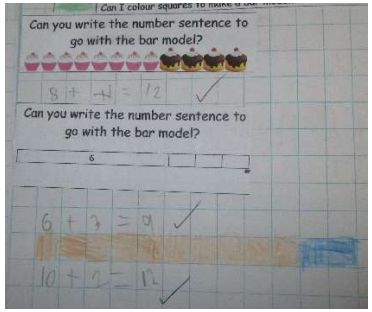
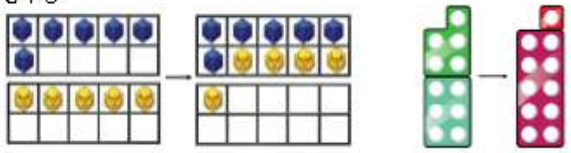
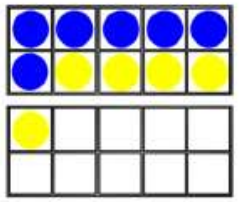
Vocab: add, plus, more than, total, sum of,

Bar model, part-part-whole, number line, tens frame, base 10, place value counters, missing numbers,

STEM sentences	Concrete (can we MAKE it?)	Pictorial (can we draw it?)	Abstract (can we write the calculation?)
_____ is a whole, _____ is a part, _____ is a part. There are _____ in total.	 Use numicon and cubes to show number bonds. Use cubes, numicon and others to add two numbers together. 	   Use pictures to add two numbers together as a group or a bar.   Introduce the bar model and the part-whole model to secure number bonds.	5 + 5 = 10 5 is a part, 5 is a part, the whole is ten. 

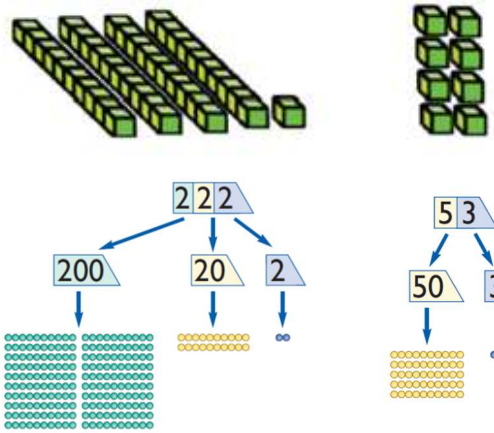
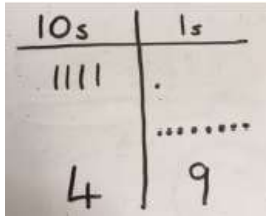
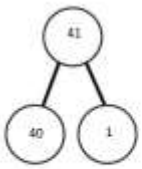
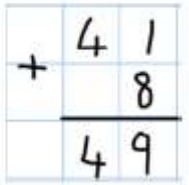
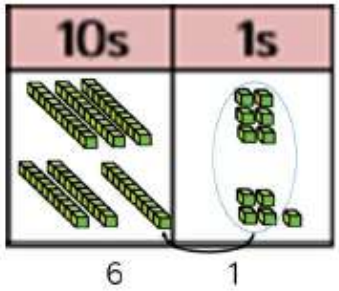
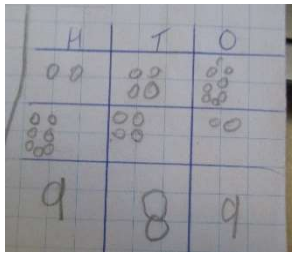
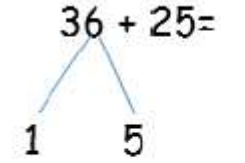
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First… Then… Now… E.g. **First** there were 4 children on the bus, **then** 3 children got on, **Now** there are 7 children on the bus. (this will help with the inverse relationship and missing number)	**Counting on using number lines** using cubes or Numicon.  How to use a hundred square…  	A bar model which encourages the children to count on rather than count all.  	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 
I need ____ to make ten. I have ____ left over. 10 + ___ is _____.	**Regrouping to make 10;** using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4

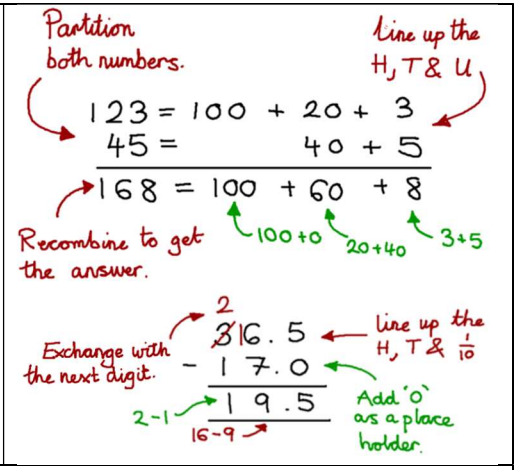
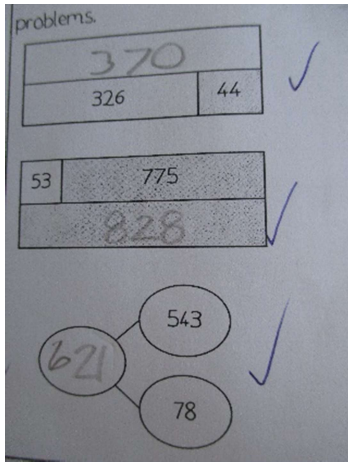
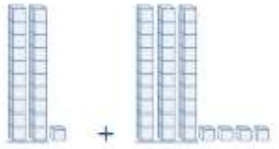
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	TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent base 10 e.g. lines for tens and dots for ones.  	41 + 8  1 + 8 = 9 40 + 9 = 49  2 + 3 = 5 20 + 50 = 70 200 + 0 = 200 200 + 70 + 5 = 275
	TO + O using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  36 +25 ___	Children to represent the base 10 or place value counters, in a place value chart 	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 Formal method: +25 61 1

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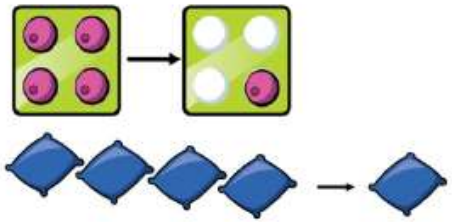
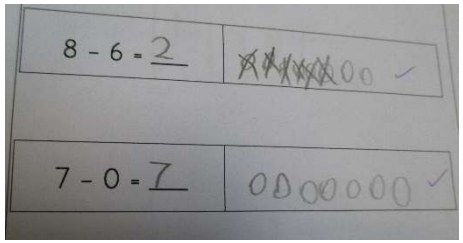
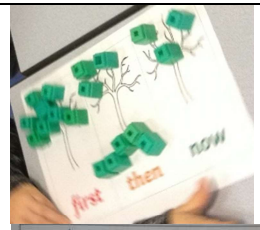
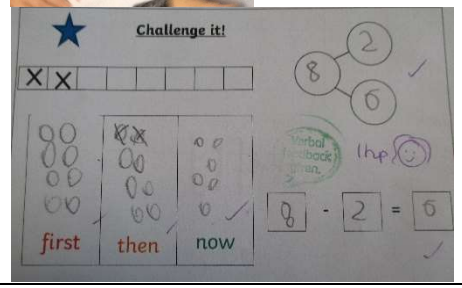
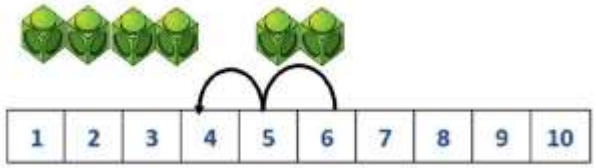
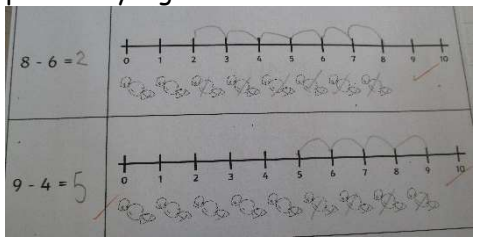
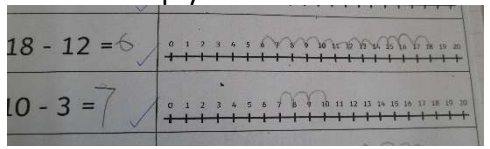
			
	Conceptual variation; different ways to ask children to solve 21 + 34		
		Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total? 21 + 34 = 55. Prove it	21 +34 **21 + 34 =** □ = 21 + 34 Calculate the sum of twenty-one and thirty-four.	 Missing digit problems: 

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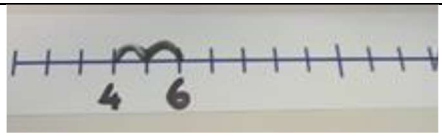
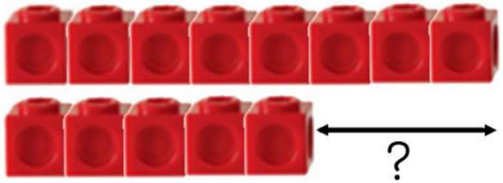
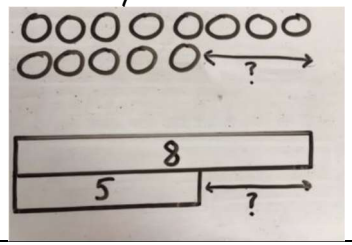
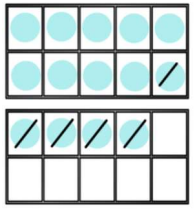
Subtraction

Vocab: take away, less than, the difference, subtract, minus, fewer, decrease, exchange

STEM sentences	Concrete	Pictorial	Abstract
First… Then… Now… e.g. **First** there were 4 children in the car, **then** 1 child got out, **Now** there are 3 children in the car.	**Physically taking away objects from a whole** (tens frame, numicon, cubes etc…) 4 - 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	 
The whole is _____. The part we are taking away is _____. Start on _____ and count back _____.	**Counting back** (using number lines or tracks) 6-2= Children start at 6 and count back 2 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

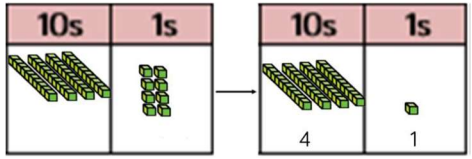
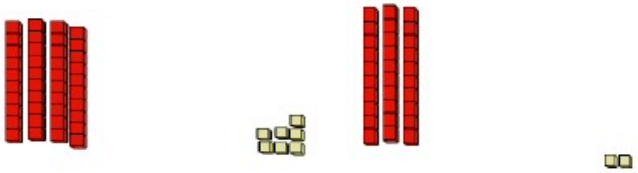
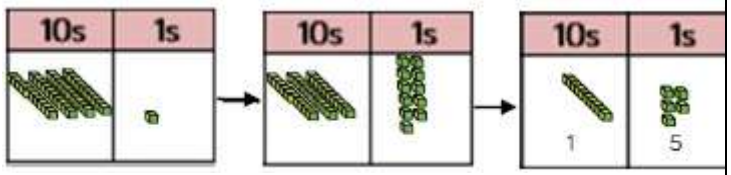
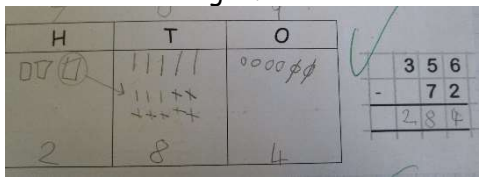
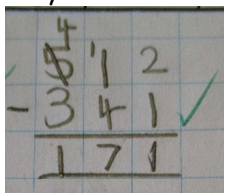
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	Finding the difference (using cubes, Numicon or other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 – 6 = 8 – 5 = 7 – 4 = have the same difference.
		Children to present the ten frame pictorially and discuss what they did to make 10. 	When children are confident they can draw this themselves. Remind them to show order and uniform to help them.

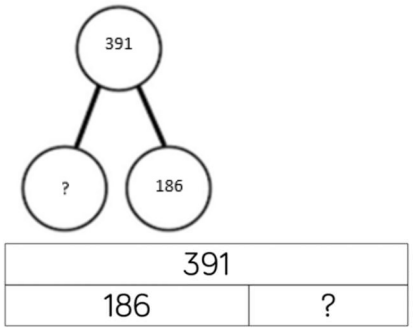
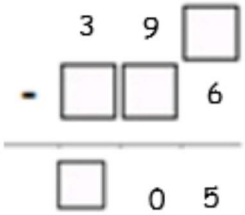
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	Column method using base 10 or place value counters. 48-7  	Children to represent the base 10 pictorially. 35 – 23 =  draw the 3 tens and 5 ones cross out the ones cross out the tens look how much remains	Encourage children to use mental strategies
		 	40 - 30 = 10 7 – 2 = 5 10 + 5 = 15
	Column method using base 10 or place value counters and having to exchange. 41 – 26  41 Here 1 ten has been exchanged for 10 ones Answer	Represent the place value counters pictorially; remembering to show what has been exchanged.  	Formal column method. Children must understand what has happened when they have crossed out digits. 

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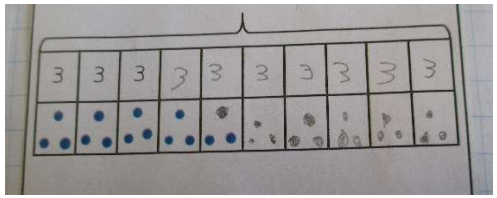
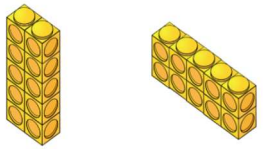
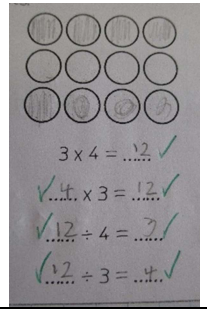
	Conceptual variation; different ways to ask children to solve 391 – 186			
		Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	☐ = 391 – 186 391 -186 ——— What is 186 less than 391?	Missing digit calculations 

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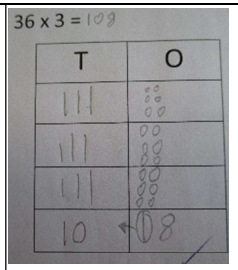
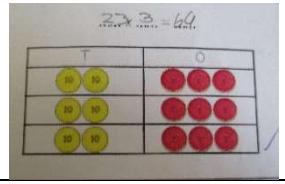
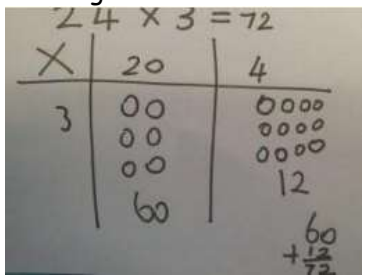
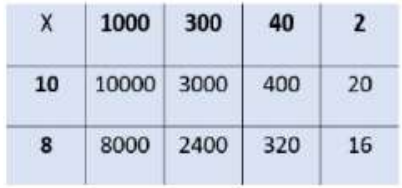
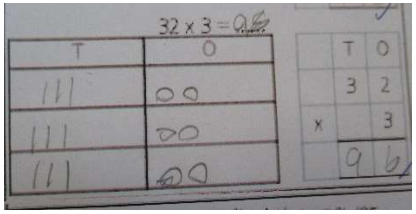
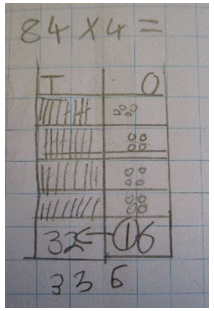
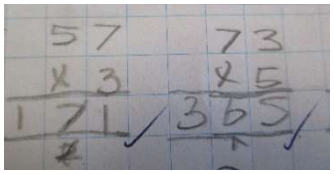
Multiplication

Vocab: double, times, multiplied by, the product of, groups of, lots of, equal groups, exchange

STEM sentences	Concrete	Pictorial	Abstract
We are counting in multiples of ____ so we count every ____. There are ____ in each group. There are ____ groups. We have to add ____ _____ times.	Repeated grouping/repeated addition 7 × 2 2 + 2 + 2 + 2 + 2 + 2 + 2 = There are 7 equal groups, with 2 in each group.  	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
____ lots of ____ is the same as ____ lots of ____.	**Use arrays to illustrate commutativity** counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	 Children to represent the arrays pictorially.	Children to be able to use an array to write a range of calculations e.g. • 10 = 2 × 5 • 5 × 2 = 10 • 2 + 2 + 2 + 2 + 2 = 10 • 10 = 5 + 5
____ can be partitioned into	To first introduce the grid method, use Base 10 before moving towards a more compact method.	Children can represent the work they have done with place value counters in a way that they understand. They can draw	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

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____ and ____. ____ lots of ___ ones is ____. ____ lots of ____ tens is ____. ____ ones add ____ tens is ____.	 36 x 3 = Make 36 on each row, there are 3 rows. Add up each column, starting with the ones and exchange if needed. Move on to Place Value Counters to show how we are finding groups of a number. We are multiplying by 3 so need 3 rows. 	the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 	 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 
____ ones times ____ ones is ___ ones. ____ ones times ____ tens is ____ tens. Because we are multiplying by ten, we need to add in a zero as a place value holder.	**Formal column method** with place value counters (base 10 can also be used.) 32 x 3 	Children to represent the base 10 or place value counters pictorially. 	Formal Written Method 

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<table>
<tr>
<td>We cannot have more than one digit in any place value column, so we need to exchange ___ ones as ____ ten</td>
<td colspan="3">When children start to multiply 3d × 3d and 4d × 2d etc…, they should be confident with the abstract:

To get 744 children have solved 6 × 124.

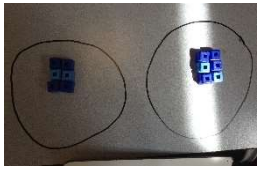
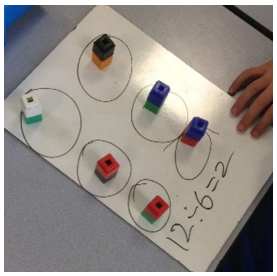
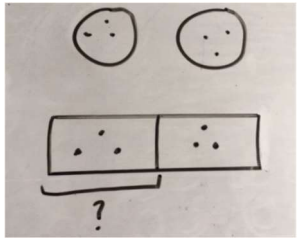
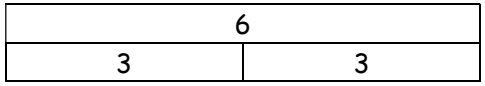
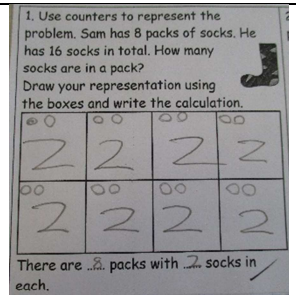
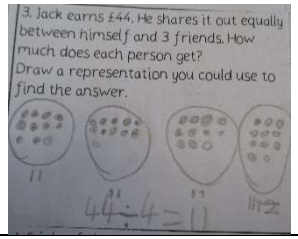
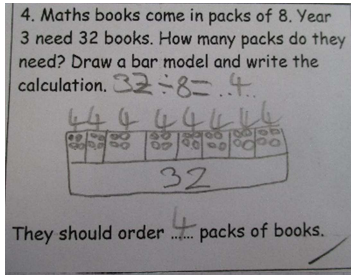
To get 2480 they have solved 20 × 124.</td>
<td>
<table>
<tr><td></td><td>1</td><td>2</td><td>4</td></tr>
<tr><td>×</td><td></td><td>2</td><td>6</td></tr>
<tr><td></td><td>7
1</td><td>4
2</td><td>4</td></tr>
<tr><td>2</td><td>4</td><td>8</td><td>0</td></tr>
<tr><td>3</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>1</td><td></td><td></td></tr>
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Answer: 3224
</td>
</tr>
<tr>
<td></td>
<td colspan="4">Conceptual variation; different ways to ask children to solve 23 x 6</td>
</tr>
<tr>
<td></td>
<td>
<table>
<tr><td>23</td><td>23</td><td>23</td><td>23</td><td>23</td><td>23</td></tr>
</table>
?
</td>
<td>Mae had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?
With the counters prove that 23 x 6 = 138</td>
<td>Find the product of 6 and 23
6 × 23 = __
__ = 23 × 6 = ___   6 × 23 = ___</td>
<td>What is the calculation?
What is the product?
<table>
<tr><th>100s</th><th>10s</th><th>1s</th></tr>
<tr><td></td><td>●●●●●
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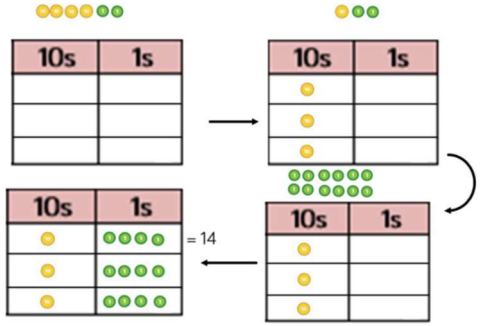
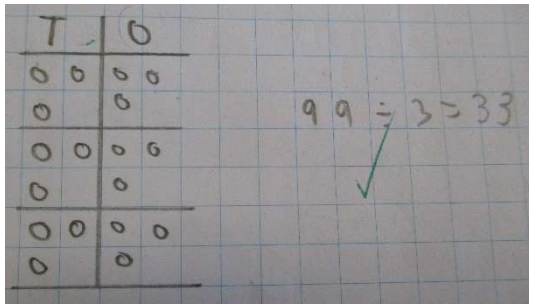
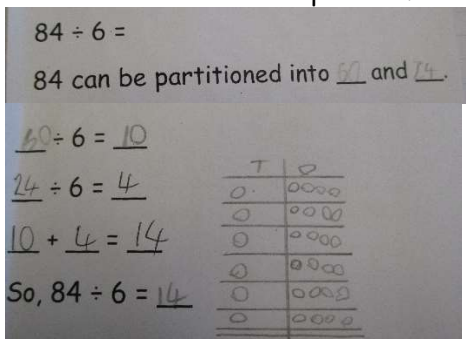
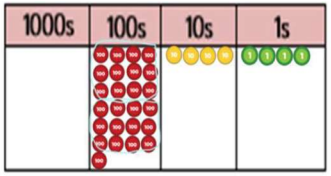
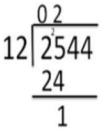
Division

Vocab: share, group, divide, divided by, half, divisor, dividend, quotient, remainder, exchange

STEM sentences	Concrete	Pictorial	Abstract
____ shared equally between ____ is _____	**Sharing** a range of objects 12 ÷ 2 =  	Represent the **sharing** pictorially 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.
	Sharing – using concrete methods  We move on to use formal jottings to help us solve the division. These can be in the form of sharing circles (squares!) 		Children can use a bar model to help represent the division.

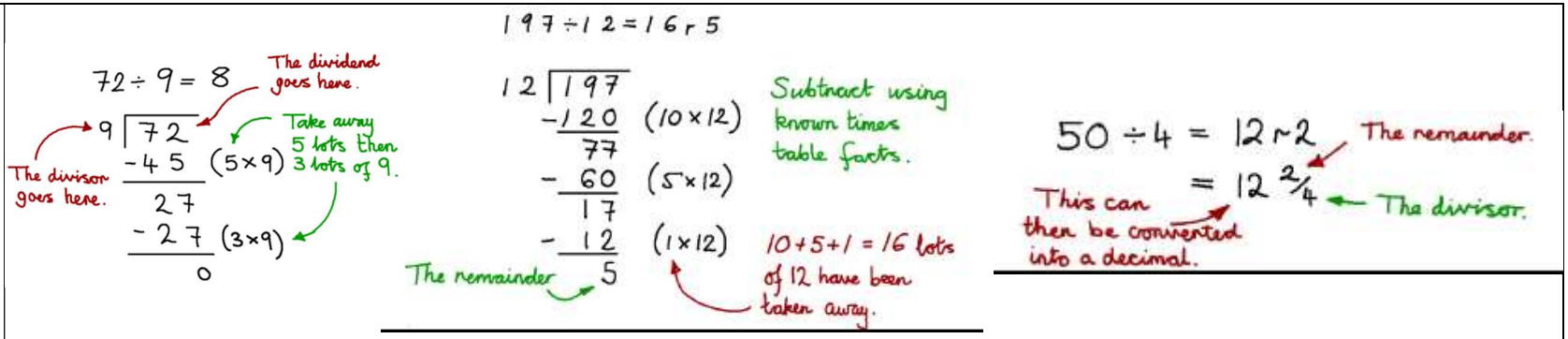
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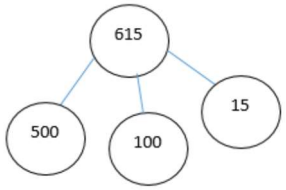
In division, we start from the largest place value column. We start from the right. ___ is __ tens and ___ ones. ____ tens divided by ____ is ____. _____ ones divided by _____ is _____. ____ add ____ is _____.	**Sharing** using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 
	6. How many groups of 5 ones can you make with 15 ones?		
	Long division using place value counters 2544 ÷ 12  We can't group 2 thousands into groups of 12 so will exchange them.  We can group 24 hundreds into groups of 12 which leaves with 1 hundred. 		

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Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division? 	I have £615 and share it equally between 5 bank accounts. How much will be in each bank account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5\overline{)615}$ 615 ÷ 5 = ___ ___ = 615 ÷ 5	What is the calculation? What is the answer? 

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Bloom's Taxonomy Question Stems Math

Remembering…questions focus on clarifying, recalling, naming, and listing
○ Which illustrates? ○ Write…in standard form… ○ What is the correct way to write the number of…in word form?
Organizing…questions focus on arranging information, comparing similarities/differences, classifying, and sequencing
○ Which shows…in order from…? ○ What is the order…? ○ Which is the difference between a …and a…? ○ Which is the same as …? ○ Express…as a…?
Applying…questions focus on prior knowledge to solve a problem
○ What was the total? ○ What is the value of? ○ How many…would be needed for…? ○ Solve… ○ Add/Subtract… ○ Find… ○ Evaluate… ○ Estimate… ○ Graph…

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Analyzing…questions focus on examining parts, identifying attributes/relationships/patterns, and main idea
○ Which tells? ○ If the pattern continues… ○ Which could? ○ What rule explains/completes…this pattern? ○ What is/are missing? ○ What is the best estimate for…? ○ Which shows…? ○ What is the effect of…?
Generating…questions focus on producing new information, inferring, predicting, and elaborating with details
○ What number does…stand for? ○ What is the probability…? ○ What are the chances…? ○ What effect…?
Integrating…questions focus on connecting/combining/summarizing information, and restructuring existing information to incorporate new information
○ How many different…? ○ What happens to…when…? ○ What is the significance of…? ○ How many different combinations…? ○ Find the number of…, …, and … in the figure below.

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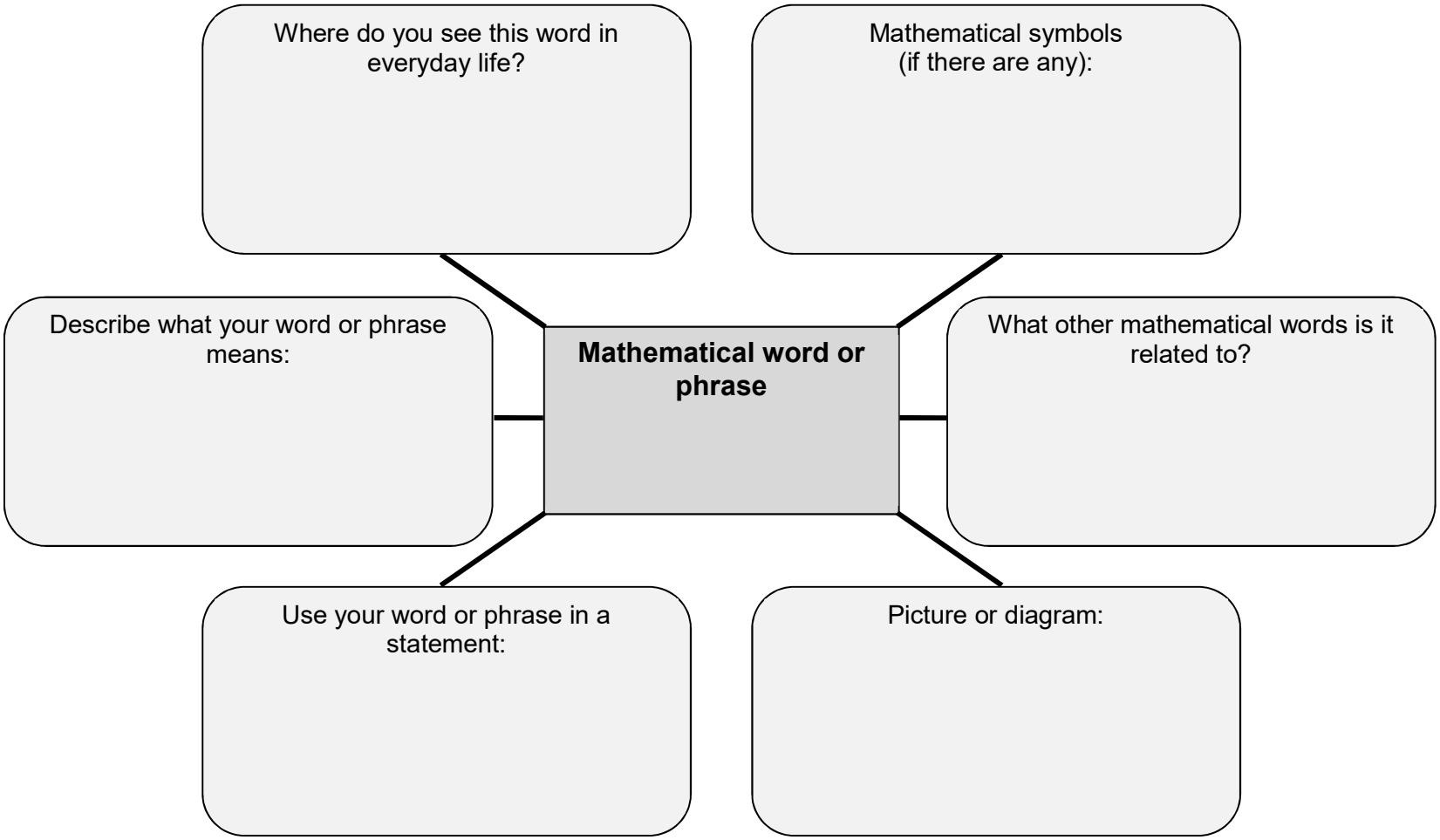
Evaluating…questions focus on reasonableness and quality of ideas, criteria for making judgments, and confirming accuracy of claims
○ Which most accurately…? ○ Which is correct? ○ Which statement about…is true? ○ What are the chances…? ○ Which would best…? ○ Which would…the same…? ○ Which statement is sufficient to proven…?

Vocabulary of reasoning…	Prompts to guide children's reasoning:
• It could be….because…. • It can't be….because…. • It won't work, because… • If….then…. • It would only work if…. • So…. • In that case…. • It follows that…. • It will/won't work when….	• What can you work out (from the information)? • If you know that, what else do you know? • Can you tell me what your thinking is? • Shall we test that? • Does it work? • Do you still think it is…? • Do you agree that…? • Why is this bit important? • So, what must it be?

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Talk in your group about what your word or phrase means.
Work together to agree the best way to complete each section of your poster.



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	8 Problem Solving Strategies
1	Act it out – make a model
2	Look for patterns
3	Guess and check – Trial and amend
4	Systematic – trial by improvement
5	Simplify the problem
6	Work backwards – use inverse
7	Make a list or table
8	Algebraically – write an equation

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Problem solving strategies		
Nursery	Year 1	Reception
Act it out / make a model: Use various resources or equipment to act out the problem and aid the solution **Look for patterns**: Identify things that happen again, repeating patterns or sequences	**Act it out / make a model**: Use various resources or equipment to act out the problem and aid the solution **Look for patterns**: Identify things that happen again, repeating patterns or sequences **Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go.	**Act it out / make a model**: Use various resources or equipment to act out the problem and aid the solution **Look for patterns**: Identify things that happen again, repeating patterns or sequences **Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go. **Trial and improvement**- a systematic approach: First estimate the starting point then refine the answer.

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Problem solving strategies		
Year 1	Year 2	Year 3
Act it out / make a model: Use various resources or equipment to act out the problem and aid the solution **Look for patterns**: Identify things that happen again, repeating patterns or sequences **Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go.	**Act it out / make a model**: Use various resources or equipment to act out the problem and aid the solution **Look for patterns**: Identify things that happen again, repeating patterns or sequences **Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go. **Trial and improvement**- a systematic approach: First estimate the starting point then refine the answer. **Simplify the problem:** break down the problem, partition the numbers	**Act it out / make a model**: Use various resources or equipment to act out the problem and aid the solution **Look for patterns**: Identify things that happen again, repeating patterns or sequences **Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go. **Trial and improvement**- a systematic approach: First estimate the starting point then refine the answer. **Simplify the problem:** break down the problem, partition the numbers **Work backwards:** use inverse operations to solve

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Problem solving strategies		
Year 4	Year 5	Year 6
Act it out / make a model: Use various resources or equipment to act out the problem and aid the solution	**Act it out / make a model**: Use various resources or equipment to act out the problem and aid the solution	**Act it out / make a model**: Use various resources or equipment to act out the problem and aid the solution
Look for patterns: Identify things that happen again, repeating patterns or sequences	**Look for patterns**: Identify things that happen again, repeating patterns or sequences	**Look for patterns**: Identify things that happen again, repeating patterns or sequences
Trial and error: least efficient strategy but a good starting point for some problems. Pick an idea and go.	**Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go.	**Trial and error**: least efficient strategy but a good starting point for some problems. Pick an idea and go.
Trial and improvement- a systematic approach: First estimate the starting point then refine the answer.	**Trial and improvement**- a systematic approach: First estimate the starting point then refine the answer.	**Trial and improvement**- a systematic approach: First estimate the starting point then refine the answer.
Simplify the problem: break down the problem, partition the numbers	**Simplify the problem:** break down the problem, partition the numbers	**Simplify the problem:** break down the problem, partition the numbers
Work backwards: use inverse operations to solve	**Work backwards:** use inverse operations to solve	**Work backwards:** use inverse operations to solve
Make a list or table: use systematic operations in a table to eliminate options	**Make a list or table:** use systematic operations in a table to eliminate options	**Make a list or table:** use systematic operations in a table to eliminate options
	Algebraically: write an equation	**Algebraically:** write an equation

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